Mathematics: analysis and approaches
Higher level
Paper 2

Name

Date: _____

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written in the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (55 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A discrete random variable has a probability distribution given in the following table.

x	3	4	5	6	7	8
$\mathrm{P}(X = x)$	0.08	a	0.30	b	0.24	0.12

(a) Given that the expected value of X is 5.62, find the value of a and the value of b. [4]

(b) Calculate the variance of X. [2]

2. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A \cup B) = 0.9$, $\mathrm{P}(A \cap B) = 0.45$ and $\mathrm{P}(A \mid B) = 0.75$.

(a) Find $\mathrm{P}(B)$. [2]

(b) Find $\mathrm{P}(A)$. [2]

(c) Hence, show that events A and B are independent. [2]

3. [Maximum mark: 5]

Consider the function $y = p + \frac{p^2}{x} + x^2$, $x \neq 0$, where p is a constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

(b) The graph of the function has a local minimum point at $(2,8)$. Find the value of p. [4]

4. [Maximum mark: 7]

A multiple choice test consists of twelve questions. Each question has four answers. Only one of the answers is correct. For each question, Emma randomly chooses one of the fours answers.

(a) Write down the expected number of questions Emma answers correctly. [1]

(b) Find the probability that Emma answers exactly four questions correctly. [2]

(c) Find the probability that Emma answers more than four questions correctly. [3]

5. [Maximum mark: 7]

It is known that two out of five cups of coffee served at Bella's Coffee Shop contain more than 100 mg of caffeine. It is also known that four out of five cups served at Bella's contain more 85 mg of caffeine.

Assuming the amount of caffeine in a cup of coffee at Bella's is modelled by a normal distribution, find the mean and standard deviation of the caffeine content in a cup of coffee served at Bella's.

6. [Maximum mark: 7]

Prove that the sum of the cubes of any two consecutive odd integers is divisible by four.

7. [Maximum mark: 5]

In the quadratic equation $7x^2 - 8x + c = 0,\ c \in \mathbb{Q}$, one root is three times the other root. Find the exact value of c.

8. [Maximum mark: 7]

Find the value of the constant term in the expansion $(x-1)^3\left(\frac{1}{2x^2}+4x\right)^6$.

9. [Maximum mark: 6]

Show that $\arctan(x+2)-\arctan(x+1)=\arctan\left(\frac{1}{x^2+3x+3}\right)$.

Do **not** write solutions on this page.

Section B (55 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 18]

The diagram below shows a circle with centre O and radius 6 cm.

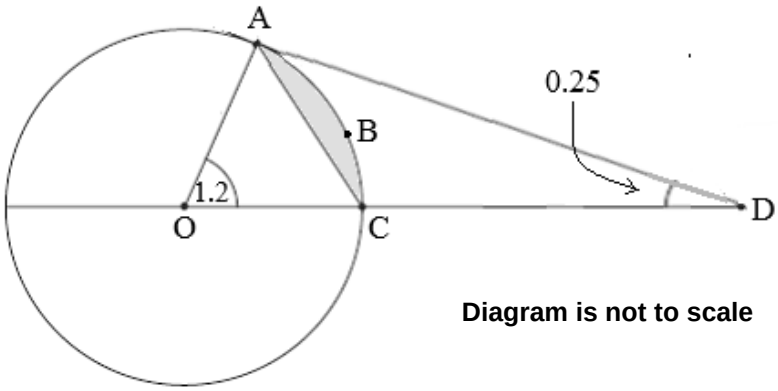

Diagram is not to scale

The points A, B and C lie on the circle. The point D is outside the circle and lies on (OC). Angle $\mathrm{AOC} = 1.2$ radians and angle $\mathrm{ADO} = 0.25$ radians.

(a) Find the area of the sector OABC. [3]

(b) Find the area of the shaded region bounded by the chord AC and the arc ABC. [4]

(c) Find AD. [3]

(d) Find OD [4]

(e) Find the area of the region ABCD. [4]

11. [Maximum mark: 21]

Consider the points $\mathrm{P}(2,-1,0)$, $\mathrm{Q}(3,0,1)$ and $\mathrm{R}(1,m,2)$, such that $m \in \mathbb{Z}, m < 0$.

(a) (i) Find the scalar product $\overrightarrow{\mathrm{QP}} \cdot \overrightarrow{\mathrm{QR}}$.

(ii) Hence, given that $\mathrm{P\hat{Q}R} = \arccos\frac{\sqrt{2}}{3}$, show that $m = -1$. [6]

(b) Determine the Cartesian equation of the plane Π containing points P, Q and R. [4]

(c) Find the **exact** area of triangle PQR. [4]

(d) (i) The line L is perpendicular to plane Π and passes through P. Find a vector equation of L.

(ii) The point $S(6,-7,2)$ lies on L. Find the volume of the pyramid PQRS. [7]

Do **not** write solutions on this page.

12. [Maximum mark: 16]

Consider the function $f(x) = \ln(1+\sin x)$. The Maclaurin series for $f(x)$ up to and including the x^4 term is $f(x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$.

(a) Show that the Maclaurin series for $g(x) = \ln(1-\sin x)$ up to and including the x^4 term is $g(x) = -x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{12} + \cdots$. [3]

(b) Use the Maclaurin series for $f(x)$ and $g(x)$ to show that the Maclaurin series for $h(x) = \ln(\cos x)$ up to and including the x^4 term is $h(x) = -\frac{x^2}{2} - \frac{x^4}{12} + \cdots$. [5]

(c) Hence, or otherwise, find the first two terms of the Maclaurin series for $q(x) = \tan x$. [4]

(d) Hence, calculate analytically (no GDC) the **exact** value of $\lim_{x\to 0}\left(\frac{\tan(x^2)}{\ln(\cos x)}\right)$. [4]